**1. The interpretation of terms**

Given a signature and -CCC , we define the interpretation of type expressions, typing contexts and terms over in .

The interpretation of type expression is defined as follows:

* , given as part of the -CCC

The interpretation of typing context is defined by induction on the length of the context:



Both type expressions and typing contexts are interpreted as objects in.

The interpretation of a well-typed term is a morphism from to . It is defined by induction on the proof of the typing judgement:

* ;
* ;
* where is given;
* ;
* ;
* ;
* ;
* ;
* where is the  
  ,-function such that and thus is a morphism from to .

**2. Lemma (Substitution)** If and are well-typed terms, then  
.

**Proof**

The proof is carried out by induction on the typing derivation of .

**Base cases**

In this case, is empty. Therefore, should be either or .

If is , then the type of should be and

has type and the morphism of type is unique. Then,   
.

Hence, .

If is , then

where is given

The morphism of type is already given which is . Then,   
.

Hence, .

**Inductive steps:**

**(1) Projection**

By the inductive hypothesis,

**(2)**

By the inductive hypothesis,

( by )

Hence,

**(3) Application**

By the inductive hypothesis,

Then,

( by )

Hence,

**(4) Lambda abstraction**

By the inductive hypothesis,

By lemma ,

where .

Then,

(How???)

**3. Theorem (Soundness)** If , then every CCC satisfying also satisfies , i.e. the interpretations of and are the same morphism in every CCC satisfying .

**Proof**

It is proved by induction on equational proofs from .

The base cases include: , , , and .

The inductive steps include: , , and .

If just contains -, - and -equivalence, the soundness can be stated as follows:

Given and , with , then the interpretations of them equal, i.e. , in every CCC satisfying -equivalence.

**(1) -equivalence** If and are well-typed, with , then  
.

The interpretations of the two typing contexts are same product object, i.e.  
.

**(2) -equivalence**

.

( by )

( by )

( by CCC Substitution Lemma )

Hence, .

**(3) -equivalence**

.

( by and )

( by )

**4. Theorem (Completeness)** Let be any theory. There exists a CCC such that if satisfies, then .

**Proof**

This CCC is the one generated from :

The objects of are the types over the signature of and the arrows are equivalence classes of terms. We choose one variable of each type, and define the arrows from to using terms over the chosen free variable of type .

We write for the arrow of given by the term , i.e.  
. Then the operations in CCC can be defined as follows: